Mini Homework 3
Name: - Math 117 - Summer 2022

1) (2 points) Let V be a finite dimensional vector space and $W \subseteq V$ a subspace. Prove that W has a complimentary subspace

Solution:

2) (2 points) Let Let V be a finite dimensional vector space and $W \subseteq V$ a subspace. Recall the relation $\sim_W$ we defined on V:

$$x \sim_W y \iff x - y \in W$$

Prove that this is an Equivalence relation

Solution:

3) Let V be a vector space over $\mathbb{F}$

(a) (1 point) Let $U \subset V$ be a subspace. What is $U + U$? What is $\{0_V\} + U$?

(b) (2 points) Prove or disprove with a counterexample the following claim: If we have subspaces W_1, W_2, U of V such that

$$W_1 + U = W_2 + U$$

then we must have $W_1 = W_2$

Solution:

4) Let I = (π, π) be the open interval and consider the $\mathbb{R}$ vector space

$$\text{Diff } (I, \mathbb{R}) = \{f : \text{I} \to \mathbb{R} : f \text{ is differentiable for all t } \in \text{I}\}$$

Consider the two subsets

$$\text{Even}(\text{I}, \mathbb{R}) = \{f \in \text{Diff}(\text{I}, \mathbb{R}) : f(t) = f(-t) \text{ for all } t \in \text{I}\}$$
$$\text{Odd}(\text{I}, \mathbb{R}) = \{f \in \text{Diff}(\text{I}, \mathbb{R}) : f(t) = -f(-t) \text{ for all } t \in \text{I}\}$$

(a) (1 point) Prove that Even(I, $\mathbb{R}$), and Odd(I, $\mathbb{R}$) are subspaces

(b) (2 points) Prove that Diff (I, $\mathbb{R}$) = Even(I, $\mathbb{R}$) $\oplus$ Odd(I, $\mathbb{R}$)

Solution: